PUBLISHED BY INSTITUTE OF PHYSICS PUBLISHING FOR SISSA

RECEIVED: *May 7, 2008*
ACCEPTED: *July 28, 2008*
PUBLISHED: *August 11, 2008*

Localization and mass spectrum of matters on Weyl thick branes

Yu-Xiao Liu, Li-Da Zhang, Shao-Wen Wei and Yi-Shi Duan

Institute of Theoretical Physics, Lanzhou University,
Lanzhou 730000, P. R. China
E-mail: liuyx@lzu.edu.cn, zhangld04@lzu.cn, weishaow06@lzu.cn, ysduan@lzu.edu.cn

ABSTRACT: In this paper, we study localization and mass spectrum of various matter fields on a family of thick brane configurations in a pure geometric Weyl integrable 5-dimensional space time, a non-Riemannian modification of 5-dimensional Kaluza-Klein (KK) theory. We present the shape of the mass-independent potential of the corresponding Schrödinger problem and obtain the KK modes and mass spectrum, where a special coupling of spinors and scalars is considered for fermions. It is shown that, for a class of brane configurations, there exists a continuum gapless spectrum of KK modes with any $m^2 > 0$ for scalars, vectors and ones of left chiral and right chiral fermions. All of the corresponding massless modes are found to be normalizable on the branes. However, for a special of brane configuration, the corresponding effective Schrödinger equations have modified Pöschl-Teller potentials. These potentials suggest that there exist mass gap and a series of continuous spectrum starting at positive m^2. There are one bound state for spin one vectors, which is just the normalizable vector zero mode, and two bound KK modes for scalars. The total number of bound states for spin half fermions is determined by the coupling constant η. In the case of no coupling ($\eta = 0$), there are no any localized fermion KK modes including zero modes for both left and right chiral fermions. For positive (negative) coupling constant, the number of bound states of right chiral fermions is one less (more) than that of left chiral fermions. In both cases ($\eta > 0$ and $\eta < 0$), only one of the zero modes for left chiral fermions and right chiral fermions is bound and normalizable.

KEYWORDS: Large Extra Dimensions, Field Theories in Higher Dimensions.

Contents

1. Introduction

Suggestions that extra dimensions may not be compact [1 − 7] or large [8, 9] can provide new insights for a solution of gauge hierarchy problem [9] and cosmological constant problem [4, 7, 10]. In the framework of brane scenarios, gravity is free to propagate in all dimensions, whereas all the matter fields are confined to a 3-brane [9, 4, 6, 11, 12]. In ref. [1], an alternative scenario of the compactification has been put forward. In the brane world scenario, an important question is localization of various bulk fields on a brane by a natural mechanism. It is well known that the massless scalar field [13] and the graviton [1] can be localized on branes of different types, and that the spin 1 Abelian vector fields can not be localized on the Randall-Sundrum(RS) brane in five dimensions but can be localized on some branes in higher dimensions [14]. For spin 1/2 fermions, they do not have normalizable zero modes in five and six dimensions [13 − 19].

Recently, an increasing interest has focused on study of thick brane scenarios based on gravity coupled to scalars in higher dimensional space-time [20 − 24]. An interesting feature of these models is that one can obtain branes naturally without introducing them by hand in the action of the theory [20]. Furthermore, these scalar fields provide the "material" from which the thick branes are made of. Thirdly, for the branes with inclusion of scalar backgrounds [25], localized chiral fermions can be obtained under some conditions.

In this paper, we are interested in the thick branes based on gravity coupled to scalars in a Weyl integrable manifold [26 − 30]. In this scenario, spacetime structures with pure geometric thick smooth branes separated in the extra dimension arise. For most of these branes, there exists a single bound state which represents a stable 4-dimensional graviton and the spectrum of massive modes of Kaluza-Klein (KK) excitations is continuous without mass gap [27, 28]. This gives an very important conclusion: the claim that Weylian structures mimic classically quantum behavior does not constitute a generic feature of these geometric manifolds [27]. In ref. [30], it is shown that, for one of these branes, there exist one massless bound state (the massless 4-dimensional grivaton), one massive KK bound state and the continuum spectrum of delocalized KK modes. The mass hierarchy problem and the corrections to Newton's law in the thin brane limit was considered.

In our previous work [29], we studied localization of various matter fields on some of these pure geometrical Weyl thick branes. It is shown that, for both scalars and vectors, there exists a single bound state and a continuum gapless spectrum of massive KK states. But only the massless mode of scalars is found to be normalizable on the brane. For the massless fermions localization, there must have some kind of Yukawa coupling. The aim of the present article is to investigate localization of various matters on one of the pure geometrical thick branes obtained in refs. [26 − 28]. We will show that, in the brane model, there exist one and two discrete bound states for scalars and vectors respectively (the ground states are normalizable massless modes), and a series of continuum massive KK states for both fields. For spin 1/2 fermions, there are a finite number of bound states which depend on the coupling constant and only one of the zero modes for left and right chiral fermions is bound. The paper is organized as follows: In section 2, we first give a review of the thick brane arising from a pure geometric Weyl integrable 5-dimensional space time, which is a non-Riemannian modification of 5-dimensional KK theory. Then, in section 3, we study localization of various matter fields with spin ranging from 0 to 1 on the pure geometrical thick brane in 5 dimensions by presenting the shape of the potentials of the corresponding Schröinger equations. Finally, a brief conclusion and discussion are presented.

2. Review of Weyl thick branes

Let us start with a pure geometrical Weyl action in five dimensions — a non-Riemannian generalization of KK theory

$$S_5^W = \int_{M_5^W} \frac{d^5x\sqrt{-g}}{16\pi G_5} e^{\frac{3}{2}\omega} \left[R + 3\tilde{\xi}(\nabla\omega)^2 + 6U(\omega) \right], \tag{2.1}$$

where M_5^W is a 5-dimensional Weyl-integrable manifold specified by the pair (g_{MN}, ω), and ω is a Weyl scalar function. The Weylian Ricci tensor is given by $R_{MN} = \Gamma^P_{MN,P} - \Gamma^P_{PM,N} + \Gamma^P_{MN}\Gamma^Q_{PQ} - \Gamma^P_{MQ}\Gamma^Q_{NP}$, with $\Gamma^P_{MN} = \{^{\,P}_{MN}\} - \frac{1}{2}(\omega_{,M}\delta^P_N + \omega_{,N}\delta^P_M - g_{MN}\omega^{,P})$ the affine connections on M_5^W and $\{^{\,P}_{MN}\}$ the Christoffel symbols. The parameter $\tilde{\xi}$ is a coupling constant, and $U(\omega)$ is a self-interaction potential for ω. The Weyl action is of pure geometrical nature since the scalar field ω enters in the definition of the affine connections of the Weyl manifold. The line-element which results in a 4-dimensional Poincaré invariance of the Weyl action (2.1) is assumed as follows

$$ds_5^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \tag{2.2}$$

where $e^{2A(y)}$ is the warp factor, and y is the extra coordinate. Solutions can be found in refs. [26 − 29]. Here, we only reconsider the following one which corresponds to the self-interaction potential $U = \lambda e^{p\,\omega}$

$$e^{2A(y)} = \left[\cos\left(\sqrt{8\lambda p}(y-c)\right)\right]^{\frac{3}{2p}}, \tag{2.3}$$

$$e^{\omega} = \left[\cos\left(\sqrt{8\lambda p}(y-c)\right)\right]^{-\frac{2}{p}}. \tag{2.4}$$

where $p = 16\tilde{\xi} - 15$, λ and c are arbitrary constants. The solution results in a compact manifold along the extra dimension with range $-\frac{\pi}{2} \leq \sqrt{8\lambda p}(y-c) \leq \frac{\pi}{2}$, and describes a single Weyl thick brane. For more details, one can refer to refs. [26 – 28, 30].

3. Localization of various matters

Since gravitons can be localized on the Weyl thick brane described in previous section, it is a pertinent question to ask whether various bulk mater fields such as scalars, spin one vector fields and spin 1/2 fermions can be localized on the Weyl thick brane by means of only the gravitational interaction. We will analyzing the spectrums of various mater fields for the thick brane by present the potential of the corresponding Schrödinger equation. In order to get mass-independent potential, we will change the metric given in (2.2) to a conformally flat one

$$ds_5^2 = e^{2A}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right) \tag{3.1}$$

by performing the coordinate transformation

$$dz = e^{-A(y)}dy. \tag{3.2}$$

3.1 Spin 0 scalar field

Let us first consider the action of a massless real scalar coupled to gravity

$$S_0 = -\frac{1}{2}\int d^5x\sqrt{-g}\; g^{MN}\partial_M\Phi\partial_N\Phi. \tag{3.3}$$

By considering the conformally flat metric (3.1) the equation of motion which can be derived from (3.3) is read

$$\left(\partial_z^2 + 3(\partial_z A)\partial_z + \eta^{\mu\nu}\partial_\mu\partial_\nu\right)\Phi = 0. \tag{3.4}$$

Then, by decomposing $\Phi(x,z) = \sum_n \phi_n(x)\chi_n(z)$ and demanding $\phi_n(x)$ satisfies the 4-dimensional massive Klein-Gordon equation $(\eta^{\mu\nu}\partial_\mu\partial_\nu - m_n^2)\phi_n(x) = 0$, we obtain the equation for $\chi_n(z)$

$$\left(\partial_z^2 + 3(\partial_z A)\partial_z + m_n^2\right)\chi_n(z) = 0. \tag{3.5}$$

The full 5-dimensional action (3.3) reduces to the standard 4-dimensional action for the massive scalars, when integrated over the extra dimension under the conditions that the above equation is satisfied and the following normalization condition is obeyed

$$\int_{-\infty}^{\infty} dz\; e^{3A}\chi_m(z)\chi_n(z) = \delta_{mn}. \tag{3.6}$$

JHEP08(2008)041

By defining $\widetilde{\chi}_n(z) = e^{\frac{3}{2}A}\chi_n(z)$, the wave function (3.4) can be recast in the form of a Schrödinger equation

$$\left[-\partial_z^2 + V_0(z)\right] \widetilde{\chi}_n(z) = m_n^2 \widetilde{\chi}_n(z), \tag{3.7}$$

where m_n is the mass of the KK excitation and the effective potential is given by

$$V_0(z) = \frac{3}{2}\partial_z^2 A + \frac{9}{4}(\partial_z A)^2. \tag{3.8}$$

The potential has the same form as the case of graviton and has been discussed detailedly in [30]. Here we only give the main results. In order to map the compact y-interval onto the real z-line, we require that $0 < p \leq 3/4$, which results in that $z(y)$ is a monotonous function and $z \to \pm\infty$ when $\sqrt{8\lambda p}(y-c) \to \pm\frac{\pi}{2}$. For $0 < p < 3/4$, the limit of the potential $V_0(z)$ is zero when $z \to \pm\infty$, which results in that the Schrödinger equation (3.7) will have a continuous spectrum starting at zero and only the massless mode is bound. For the case $p = 3/4$, the limit of the potential $V_0(z)$ is the finite value $V_0(\infty) = 27\lambda/2$, and one can invert the coordinate transformation $dz = e^{-A(y)}dy$: $\cos[\sqrt{6\lambda}(y-c)] = \mathrm{sech}\left[\sqrt{6\lambda}(z-z_0)\right]$. The potential becomes a modified Pöschl-Teller potential

$$V_0(z) = \frac{9\lambda}{2}\left\{3 - 5\,\mathrm{sech}^2\left[\sqrt{6\lambda}(z-z_0)\right]\right\}. \tag{3.9}$$

For the potential, there are two bound states (see, e.g., [31]). One is the normalizable ground state

$$\widetilde{\chi}_0(z) = c_0 \mathrm{sech}^{3/2}(\sqrt{6\lambda}(z-z_0)) \tag{3.10}$$

with mass $m_0^2 = 0$ (zero-mass state), another is the normalizable exited state

$$\widetilde{\chi}_1(z) = c_1 \sinh(z)\mathrm{sech}^{3/2}(\sqrt{6\lambda}(z-z_0)) \tag{3.11}$$

with mass $m_1^2 = 12\lambda$. The continuous spectrum start with $m^2 = \frac{27}{2}\lambda$ and asymptotically turn into plane waves, which represent delocalized KK massive scalars. The potential V_0 and the mass spectrum are showed in figure 1.

3.2 Spin 1 vector field

Next we turn to spin 1 vector fields. Here we consider the action of U(1) vector fields

$$S_1 = -\frac{1}{4}\int d^5x\sqrt{-g}g^{MN}g^{RS}F_{MR}F_{NS}, \tag{3.12}$$

where $F_{MN} = \partial_M A_N - \partial_N A_M$ as usual. From this action and the background geometry (3.1), the equation motion is given by

$$\eta^{\mu\nu}\partial_\mu F_{\nu 4} = 0, \tag{3.13}$$

$$\partial^\mu F_{\mu\nu} + (\partial_z + \partial_z A)\, F_{4\nu} = 0. \tag{3.14}$$

We assume that the A_μ are Z_2-even and that A_4 is Z_2-odd with respect to the extra dimension z, which results in that A_4 has no zero mode in the effective 4D theory. Furthermore,

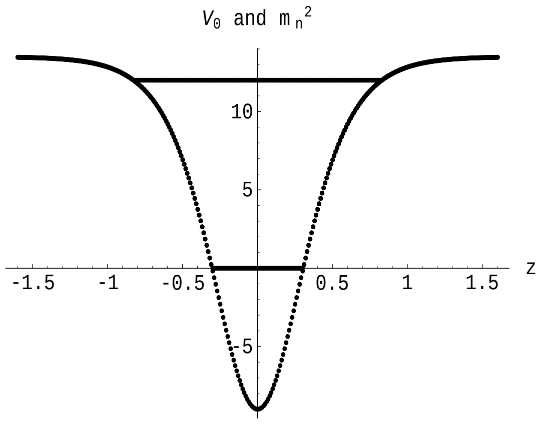

Figure 1: The shape of the potentials V_0 and the mass spectrum m_n^2 for scalars for the case $p = 3/4$. The parameter is set to $\lambda = 1$.

in order to consistent with the gauge invariant equation $\oint dz A_4 = 0$, we use gauge freedom to choose $A_4 = 0$. Under these assumption, the action (3.12) is reduced to

$$S_1 = -\frac{1}{4}\int d^4x dz \left(e^A \eta^{\mu\lambda}\eta^{\nu\rho}F_{\mu\nu}F_{\lambda\rho} - 2\eta^{\mu\nu}A_\mu \partial_z \left(e^A \partial_z A_\nu\right)\right). \tag{3.15}$$

Then, with the decomposition of the vector field $A_\mu(x,z) = \sum_n a_\mu^{(n)}(x)\rho_n(z)$, and importing the normalization condition

$$\int_{-\infty}^{\infty} dz\ e^A \rho_m(z)\rho_n(z) = \delta_{mn}, \tag{3.16}$$

the action (3.15) is read

$$S_1 = \sum_n \int d^4x \left(-\frac{1}{4}\eta^{\mu\lambda}\eta^{\nu\rho} f_{\mu\nu}^{(n)} f_{\lambda\rho}^{(n)} - \frac{1}{2}m_n^2 \eta^{\mu\nu} a_\mu^{(n)} a_\nu^{(n)}\right), \tag{3.17}$$

where $f_{\mu\nu}^{(n)} = \partial_\mu a_\nu^{(n)} - \partial_\nu a_\mu^{(n)}$ is the 4-dimensional field strength tensor, and it has been required that the $\rho_n(z)$ satisfies the equation

$$(\partial_z^2 + (\partial_z A)\partial_z + m_n^2)\rho_n(z) = 0. \tag{3.18}$$

By defining $\widetilde{\rho}_n = e^{A/2}\rho_n$, we get the corresponding Schrödinger equation for the vector field

$$\left[-\partial_z^2 + V_1(z)\right]\widetilde{\rho}_n(z) = m_n^2 \widetilde{\rho}_n(z), \tag{3.19}$$

where the potential is given by

$$V_1(z) = \frac{1}{2}\,\partial_z^2 A + \frac{1}{4}\,(\partial_z A)^2. \tag{3.20}$$

JHEP08(2008)041

The potential can be calculated as a function of y by using the coordinate transformation (3.2).

$$V_1(z(y)) = \frac{3\lambda}{8p} \cos^{3/2p-2}(\sqrt{8\lambda p}\, y) \left(9\sin^2(\sqrt{8\lambda p}\, y) - 8p\right), \tag{3.21}$$

where we have set $c = 0$. For $0 < p \leq 3/4$, $z(y)$ is a monotonous function, which implies that

$$\lim_{z \to \pm\infty} V_1(z) = \lim_{\sqrt{8\lambda p}\, y \to \pm\frac{\pi}{2}} V_1(z(y)). \tag{3.22}$$

This limit is zero for $0 < p < 3/4$ and $3\lambda/2$ for $p = 3/4$. So for the case $0 < p < 3/4$, the Schrödinger equation (3.19) will have a continuous spectrum starting at zero and only the massless mode is bound. For the case $p = 3/4$, one can invert the coordinate transformation $dz = e^{-A(y)}dy$ and explicit form of the potential is turned out to be

$$V_1(z) = \frac{3}{2}\lambda \left(1 - 3\mathrm{sech}^2(\sqrt{6\lambda}\, z)\right). \tag{3.23}$$

This potential has a minimum (negative value) $V_1(z = 0) = -3\lambda$ at the location of brane and the asymptotic behavior: $V_1(z = \pm\infty) = 3\lambda/2$, which implies that there is a mass gap. By rescaling $u = \sqrt{6\lambda}\, z$, eq. (3.19) turns into the well-known Schrödinger equation with $\nu = 1/2$ and $E_n = m_n^2/6\lambda - 1/4$

$$\left[-\partial_u^2 - \nu(\nu+1)\mathrm{sech}^2(u)\right] \widetilde{\rho}_n = E_n\, \widetilde{\rho}_n. \tag{3.24}$$

For this equation with a modified Pöschl-Teller potential, there is only one bound state, i.e., the ground state

$$\widetilde{\rho}_0(z) = \frac{(6\lambda)^{1/4}}{\sqrt{\pi}} \mathrm{sech}^{1/2}(\sqrt{6\lambda}\, z) \tag{3.25}$$

with energy $E_0 = -\nu^2 = -1/4$, which is just the normalized zero-mass mode and also shows that there is no tachyonic vector modes. The potential V_1 and the mass spectrum are showed in figure 2. The continuous spectrum start with $m^2 = \frac{3}{2}\lambda$ and asymptotically turn into plane waves, which represent delocalized KK massive vectors.

It was shown in the RS model in AdS_5 space that a spin 1 vector field is not localized neither on a brane with positive tension nor on a brane with negative tension so the Dvali-Shifman mechanism [32] must be considered for the vector field localization [13]. Here, it is turned out that a vector field can be localized on the thick brane for $0 < p \leq 3/4$ and we do not need to introduce additional mechanism for the vector field localization in the case at hand. For both cases of $0 < p < 3/4$ and $p = 3/4$, there is only one bound state which is the zero-mass mode. While for the latter case $p = 3/4$, there exist a mass gap between the ground state and the first exited state.

JHEP08(2008)041

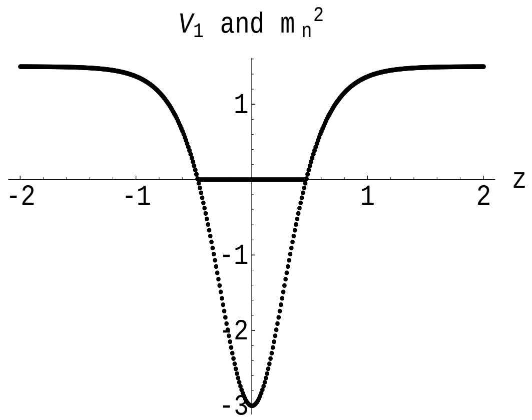

Figure 2: The shape of the potentials V_1 and the mass spectrum m_n^2 for vectors for the case $p = 3/4$. The parameter is set to $\lambda = 1$.

3.3 Spin 1/2 fermionic field

In this subsection, we will investigate whether spin half fermions can be localized on the brane. In five dimensions, fermions are four component spinors and their Dirac structure is described by $\Gamma^M = e^M_{\bar{M}}\Gamma^{\bar{M}}$ with $\{\Gamma^M, \Gamma^N\} = g^{MN}$. In our set-up, $\Gamma^M = (e^{-A}\gamma^\mu, e^{-A}\gamma^5)$, where γ^μ and γ^5 are the usual flat gamma matrices in the Dirac representation. The Dirac action of a massless spin 1/2 fermion coupled to gravity and scalar is

$$S_{1/2} = \int d^5x\sqrt{-g}\left(\bar{\Psi}\Gamma^M D_M\Psi - \eta\bar{\Psi}F(\omega)\Psi\right), \tag{3.26}$$

where the covariant derivative D_M is defined as $D_M\Psi = (\partial_M + \frac{1}{4}\omega_M^{\bar{M}\bar{N}}\Gamma_{\bar{M}}\Gamma_{\bar{N}})\Psi$ with the spin connection $\omega_M = \frac{1}{4}\omega_M^{\bar{M}\bar{N}}\Gamma_{\bar{M}}\Gamma_{\bar{N}}$. In this paper, $\bar{M}, \bar{N}, \ldots$ denote the local Lorentz indices, and $\Gamma^{\bar{M}}$ are the flat gamma matrices in five dimensions. In background (3.1), the non-vanishing components of the spin connection ω_M are

$$\omega_\mu = \frac{1}{2}(\partial_z A)\gamma_\mu\gamma_5. \tag{3.27}$$

From the Dirac action and the above equation, the equation of motion is given by

$$\left\{\gamma^\mu\partial_\mu + \gamma^5\left(\partial_z + 2\partial_z A\right) - \eta\, e^A F(\omega)\right\}\Psi = 0, \tag{3.28}$$

where $\gamma^\mu\partial_\mu$ is the Dirac operator on the brane.

We are now ready to study the above Dirac equation for 5-dimensional fluctuations, and write it in terms of 4-dimensional effective fields. Because of the Dirac structure of the fifth gamma matrix $\Gamma^5 = \gamma^5$, we expect that left- and right-handed projections of the four dimensional part to behave differently. Thus, from the equation of motion (3.28), we will search for the solutions of the general chiral decomposition

$$\Psi(x, z) = \sum_n \psi_{Ln}(x)\alpha_{Ln}(z) + \sum_n \psi_{Rn}(x)\alpha_{Rn}(z), \tag{3.29}$$

where $\psi_{Ln}(x)$ and $\psi_{Rn}(x)$ are the left-handed and right-handed components of a 4-dimensional Dirac field, they are a fixed basis and the ψ_{Ln} and ψ_{Rn} are dynamical, and the sum over n can be both discrete and continuous. To obtain the defining equations for the basis functions $\psi_{Ln}(x)$ and $\psi_{Rn}(x)$, we assume that $\psi_L(x)$ and $\psi_R(x)$ satisfy the 4-dimensional massive Dirac equations $\gamma^\mu\partial_\mu\psi_{Ln}(x) = m_n\psi_{R_n}(x)$ and $\gamma^\mu\partial_\mu\psi_{Rn}(x) = m_n\psi_{L_n}(x)$. Then $\alpha_{Ln}(z)$ and $\alpha_{Rn}(z)$ satisfy the following coupled eigenvalue equations

$$\left\{\partial_z + 2\partial_z A + \eta\, e^A F(\omega)\right\}\alpha_{Ln}(z) = \quad m_n\alpha_{Rn}(z), \tag{3.30a}$$

$$\left\{\partial_z + 2\partial_z A - \eta\, e^A F(\omega)\right\}\alpha_{Rn}(z) = -m_n\alpha_{Ln}(z). \tag{3.30b}$$

In order to obtain the standard four dimensional action for the massive chiral fermions, we need the following orthonormality conditions

$$\int_{-\infty}^{\infty} e^{4A}\alpha_{Lm}\alpha_{Rn}dz = \delta_{LR}\delta_{mn}. \tag{3.31}$$

for α_{L_n} and α_{R_n}.

By defining $\widetilde{\alpha}_{Ln} = e^{2A}\alpha_{Ln}$, we get the Schrödinger equation for left chiral fermions

$$[-\partial_z^2 + V_L(z)]\widetilde{\alpha}_{Ln} = m_n^2\widetilde{\alpha}_{Ln} \tag{3.32}$$

with the effective potential

$$V_L(z) = e^{2A}\eta^2F^2(\omega) - e^A\eta\;\partial_z F(\omega) - (\partial_z A)e^A\eta F(\omega). \tag{3.33}$$

For localization of left chiral fermions around the brane, the effective potential $V_L(z)$ should have a minimum at the brane. Furthermore, we also demand a symmetry for $V_L(z)$ about the position of the brane. This requires $F(\omega(z))$ to be an odd function of z. Since $\omega(z)$ is a even function, we set $F(\omega(z)) = e^{\frac{1}{2}p\,\omega(z)}\partial_z\omega(z)$ as an example. Here, we face the difficulty again that for general p we can not solve the function $z(y)$ in an explicit form. But we can write the potential as a function of y:

$$\begin{aligned} V_L(z(y)) &= \eta e^{2A}\left(\eta F^2 - \partial_y F - F\partial_y A\right) \\ &= \frac{4\eta\lambda}{p}\cos^{-2+3/2p}(\sqrt{8p\lambda}\;y)\left[(8\eta+3)\sin^2(\sqrt{8p\lambda}\;y) - 4p\right], \end{aligned} \tag{3.34}$$

where we have set $c = 0$. For $0 < p < 3/4$, $z(y)$ is a monotonous function, and this potential has the asymptotic behavior: $V_L(z = \pm\infty) = 0$ and $V_L(z = 0) = -16\eta\lambda$. For $\eta > 0$, this in fact is a volcano type potential [33, 34]. This means that the potential provides no mass gap to separate the fermion zero mode from exited KK modes. The potential for right chiral fermions can be obtained by the replacement $\eta \longrightarrow -\eta$ from eq. (3.34). The shape of the potentials V_L and V_R for left and right chiral fermions for the case $0 < p < 3/4$ is plotted in figure 3 in y and z coordinates.

Following, we mainly discuss the case $p = 3/4$, for which one can invert the coordinate transformation $dz = e^{-A(y)}dy$ and get the explicit form of the potential for left chiral fermion

$$V_L(z) = \frac{16\eta\lambda}{3}\left[8\eta - (8\eta+3)\mathrm{sech}^2(\sqrt{6\lambda}\;z)\right]. \qquad \left(p = \frac{3}{4}\right) \tag{3.35}$$

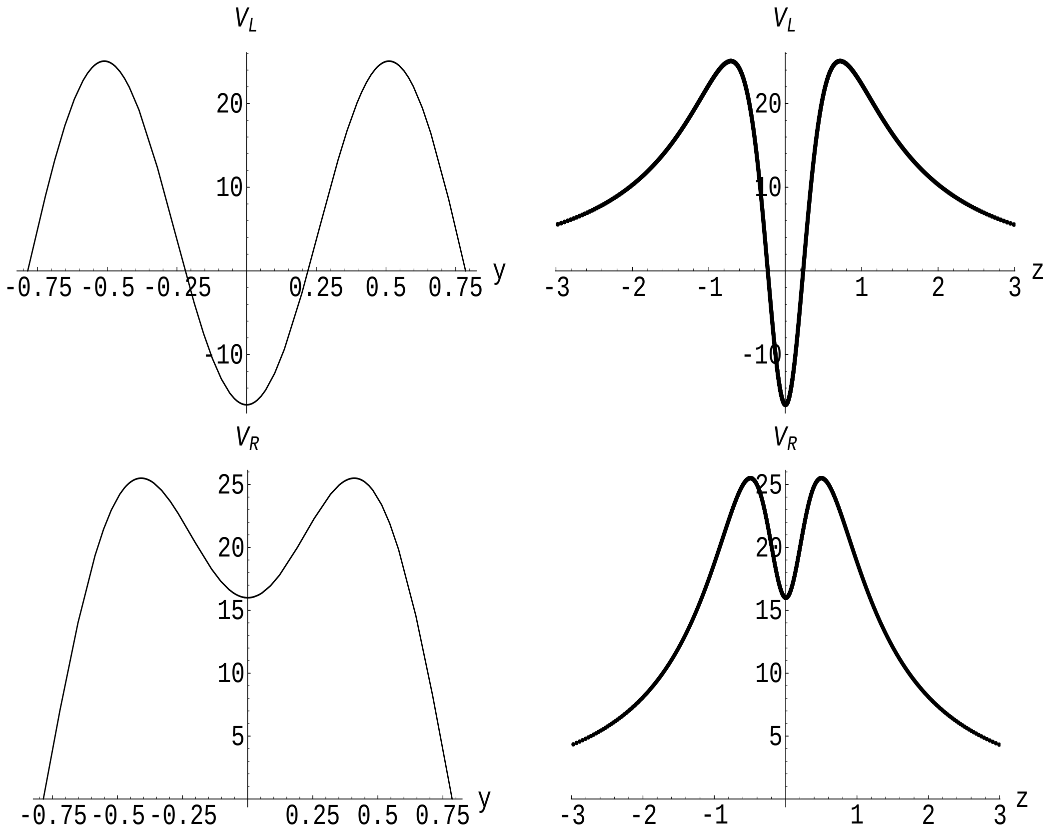

Figure 3: The shape of the potentials V_L and V_R for left and right chiral fermions for the case $0 < p < 3/4$ in y and z coordinates. The parameters are set to $p = 1/2$, $\eta = 1$ and $\lambda = 1$.

For right chiral fermion, the corresponding potential can be written out easily by replacing $\eta \to -\eta$ from above equation

$$V_R(z) = \frac{16}{3}\eta\lambda\left[8\eta - (8\eta - 3)\mathrm{sech}^2(\sqrt{6\lambda}\, z)\right], \qquad \left(p = \frac{3}{4}\right) \tag{3.36}$$

and the value at $y = 0$ is given by

$$V_R(0) = -V_L(0) = 16\eta\lambda. \tag{3.37}$$

Both the two potentials have the asymptotic behavior: $V_{L,R}(z = \pm\infty) = 128\eta^2\lambda/3 > 0$. But for a given coupling constant η, the values of the potentials at $z = 0$ are opposite. The shape of the above two potentials is shown in figure 4 for different values of positive η.

For positive η, only the potential for left chiral fermions has a negative value at the location of the brane, which can trap the left chiral fermion zero mode solved from (3.30a) by set $m_0 = 0$:

$$\widetilde{\alpha}_{L0} = \left[\frac{\sqrt{6\lambda}\,\Gamma(\frac{8\eta}{3} + \frac{1}{2})}{\sqrt{\pi}\,\Gamma(\frac{8\eta}{3})}\right]^{\frac{1}{2}} \cosh^{-\frac{8\eta}{3}}(\sqrt{6\lambda}\, z). \qquad (\eta > 0) \tag{3.38}$$

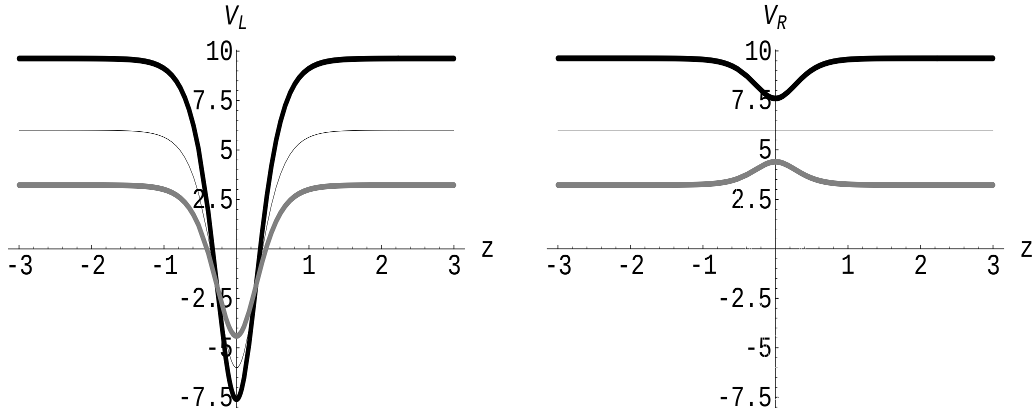

Figure 4: The shape of the potentials V_L and V_R for left and right chiral fermions for the case $p = 3/4$. The parameters are set to $\lambda = 1$, and $\eta = 3/8$ for black thin lines, $\eta = 3/8 + 0.1$ for black thick lines and $\eta = 3/8 - 0.1$ for gray thick liens.

The zero mode (3.38) represents the lowest energy eigenfunction of the Schrödinger equation (3.32) since it has no zeros. The general bound states for the potential (3.35) can be obtained by using the traditional recipe of transforming the stationary Schrödinger equation into an hypergeometric equation

$$\widetilde{\alpha}_{Ln} = \cosh^{1+\frac{8}{3}\eta}(\sqrt{6\lambda}\, z) \left[d_n f_1 + g_n \sinh(\sqrt{6\lambda}\, z) f_2\right], \tag{3.39}$$

where d_n and g_n are normalization constants and f_1 and f_2 are the hypergeometric functions

$$\begin{aligned} f_1 &= {}_2F_1\left(a_n, b_n; \frac{1}{2}; -\sinh^2(\sqrt{6\lambda}\, z)\right), \\ f_2 &= {}_2F_1\left(a_n + \frac{1}{2}, b_n + \frac{1}{2}; \frac{3}{2}; -\sinh^2(\sqrt{6\lambda}\, z)\right) \end{aligned} \tag{3.40}$$

with the parameters a_n and b_n given by

$$a_n = \frac{1}{2}(n+1), \tag{3.41}$$

$$b_n = \frac{8}{3}\eta - \frac{1}{2}(n-1). \tag{3.42}$$

The corresponding mass spectrum of the bound states is

$$m_n^2 = 2\lambda(16\eta - 3n)n. \quad \left(\eta > 0,\ n = 0, 1, 2, \ldots < \frac{8}{3}\eta\right) \tag{3.43}$$

It is turned out that the state for $n = 0$ always belongs to the spectrum of $V_L(z)$, which is just the zero mode with $m_0 = 0$. Since the ground state has the lowest mass square $m_0^2 = 0$, there is no tachyonic left chiral fermion modes. Suppose $N_L - 1$ is the biggest possible value of n in (3.43), then the number of bound states for left chiral fermions is N_L. If $0 < \eta \leq 3/8$, there is just one bound state ($N_L = 1$), i.e., the zero mode, and for which we have $a_0 = 1/2, b_0 = 8\eta/3 + 1/2, d_0 \neq 0, g_0 = 0$, the corresponding normalized wave function turns out to be the zero mode given in (3.38). In order for the left chiral fermion potential to have at least one bound exited state ($N_L \geq 2$), the condition $\eta > 3/8$ is needed. It is interesting to note that (3.35) is a modified transparent potential (i.e. the reflexion coefficient is equal to zero) when $\eta = 3k/8$ and $k = 1, 2, \ldots$ [35]

$$V_L(z) = 6\lambda\Big[k^2 - k(k+1)\text{sech}^2(\sqrt{6\lambda}\, z)\Big], \quad \left(\eta = \frac{3k}{8}\right) \tag{3.44}$$

and there are k bound states ($N_L = k$) with mass spectrum $m_n^2 = 6\lambda(2k - n)n$.

In the case $\eta > 0$, the potential for right chiral fermions is positive near the location of the brane, which shows that it can not trap the right chiral zero mode. For the special value $\eta = 3/8$, the potential of right chiral fermions is a positive constant

$$V_R(z) = 6\lambda. \quad (\eta = 3/8) \tag{3.45}$$

For the case $0 < \eta \leq 3/8$, we have $V_R(0) \geq V_R(\pm\infty) > 0$, which shows that there is no any bound state for the potential of right chiral fermions. For the case $\eta > 3/8$, $0 < V_R(0) < V_R(\pm\infty)$, there is a potential barrier which indicates that there may be some bound states, but none of them is zero mode. The corresponding mass spectrum is

$$m_n^2 = 2\lambda[16\eta - 3(n+1)](n+1). \tag{3.46}$$
$$\left(\eta > \frac{3}{8},\ n = 0, 1, 2, \ldots < \frac{8}{3}\eta - 1\right)$$

The number of bound states of right chiral fermions N_R is one less than that of left chiral fermions N_L, i.e., $N_R = N_L - 1$. If $0 < \eta \leq 3/8$, there is only one left chiral fermion bound state. If $\eta > 3/8$, there are $N_L(N_L \geq 2)$ left chiral fermion bound states and $N_L - 1$ right chiral fermion bound states. The ground state for right chiral fermion is

$$\widetilde{\alpha}_{R0} = \left[\frac{\sqrt{6\lambda}\,\Gamma(\frac{8\eta}{3} - \frac{1}{2})}{\sqrt{\pi}\,\Gamma(\frac{8\eta}{3} - 1)}\right]^{\frac{1}{2}} \cosh^{1-\frac{8\eta}{3}}(\sqrt{6\lambda}\, z), \qquad \left(\eta > \frac{3}{8}\right) \tag{3.47}$$

which is not any more zero mode because the mass is decided by $m_0^2 = 2\lambda(16\eta - 3) > 6\lambda > 0$. This conclusion can be seen clearly from the shape of V_R in figure (4). When $\eta = 3k/8$ and $k = 2, 3, \ldots$, the potential (3.36) is

$$V_R(z) = 6\lambda\Big[k^2 - k(k-1)\text{sech}^2(\sqrt{6\lambda}\, z)\Big], \qquad \left(\eta = \frac{3k}{8}\right) \tag{3.48}$$

and there are $k - 1$ bound states ($N_R = k - 1$) with mass spectrum $m_n^2 = 6\lambda[2k - (n+1)](n+1)$. In figures 5 and 6 we plot the shape of left and right chiral fermions and the mass spectrum for different values of η. For the case $\lambda = 1, \eta = 5 \times \frac{3}{8}$, there are 5 bound states for the left fermions and 4 bound states for the right ones and the mass spectra are

$$m_{Ln}^2 = \{0, 54, 96, 126, 144\} \cup [150, \infty), \tag{3.49}$$
$$m_{Rn}^2 = \{\quad 54, 96, 126, 144\} \cup [150, \infty). \tag{3.50}$$

JHEP08(2008)041

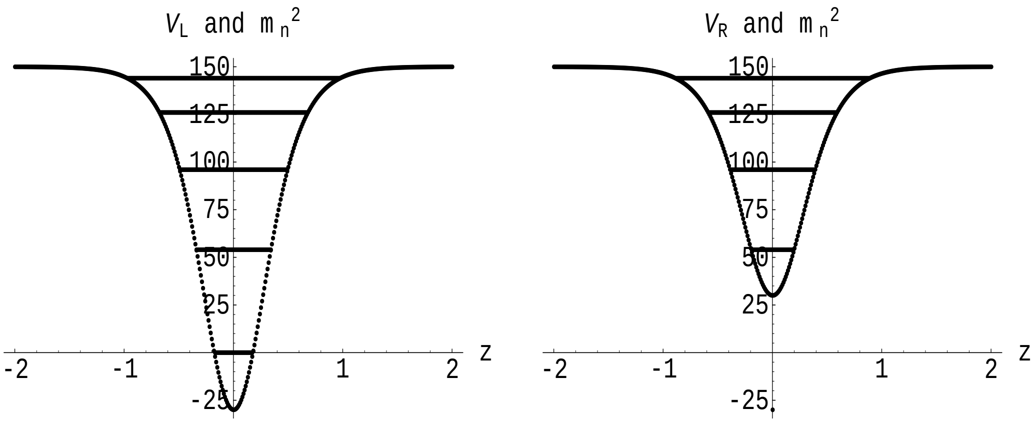

Figure 5: The shape of the potentials V_L, V_R and the mass spectrum m_n^2 for left and right chiral fermions for the case $p = 3/4$. The parameters are set to $\eta = 5 \times 3/8$ and $\lambda = 1$.

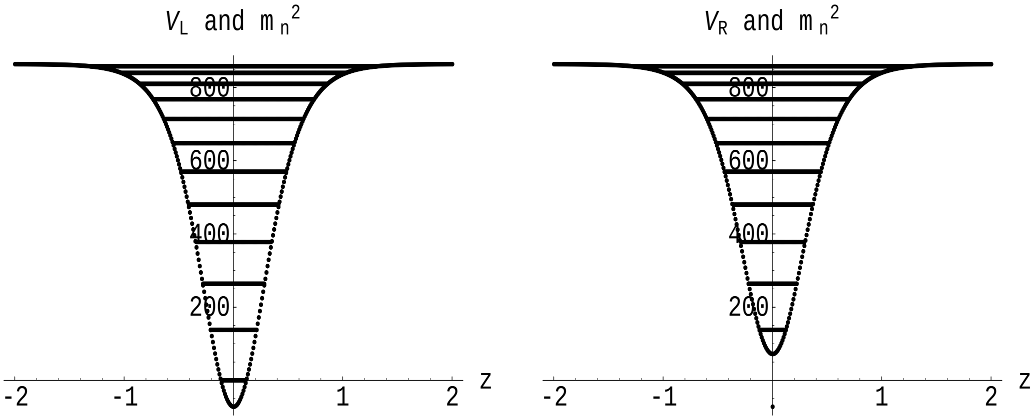

Figure 6: The shape of the potentials V_L, V_R and the mass spectrum m_n^2 for left and right chiral fermions for the case $p = 3/4$. The parameters are set to $\eta = 12 \times 3/8$ and $\lambda = 1$.

For the case $\lambda = 1, \eta = 12 \times \frac{3}{8}$, there are 12 and 11 bound states for the left and the right fermions respectively and the mass spectra are

$$m_{Ln}^2 = \{0, 138, 264, 378, 480, 570, 648, 714, 768, 810, 840, 858\} \cup [864, \infty), \tag{3.51}$$

$$m_{Rn}^2 = \{\quad 138, 264, 378, 480, 570, 648, 714, 768, 810, 840, 858\} \cup [864, \infty). \tag{3.52}$$

But for the case of negative η, things are opposite and only the right chiral zero mode

$$\widetilde{\alpha}_{R0} = \left[\sqrt{\frac{6\lambda}{\pi}} \frac{\Gamma(\frac{1}{2} - \frac{8\eta}{3})}{\Gamma(-\frac{8\eta}{3})}\right]^{\frac{1}{2}} \cosh^{\frac{8\eta}{3}}(\sqrt{6\lambda}\, z) \qquad (\eta < 0) \tag{3.53}$$

can be trapped on the brane. For arbitrary $\eta \neq 0$, the two potentials suggest that there exist mass gap (at least for one of them) and a continuous spectrum of KK modes with positive $m^2 > 0$ (for both of them), which are same as the cases of scalars and vectors obtained in the section. It is worth noting that, in the case of no coupling ($\eta = 0$), both the two potentials for left and right chiral fermions are vanish, and hence there are no any localized fermion KK modes including zero modes.

Localization of fermions in general spacetimes has been studied for example in [25]. In ref. [36], Melfo *et al* showed that only one massless chiral mode is localized in double walls and branes interpolating between different AdS_5 spacetimes whenever the wall thickness is keep finite, while chiral fermionic modes cannot be localized in dS_4 walls embedded in a M_5 spacetime. Localizing the fermionic degrees of freedom on branes or defects requires us to introduce other interactions but gravity. Recently, Parameswaran *et al* studied fluctuations about axisymmetric warped brane solutions in 6-dimensional minimal gauged supergravity and proved that, not only gravity, but Standard Model fields could be described by an effective 4-Dimensional theory [37]. Moreover, there are some other backgrounds such as gauge field [38], supergravity [39] and vortex background [40, 41] could be considered. The topological vortex coupled to fermions may result in chiral fermion zero modes [42]. More recently, Volkas *et al* had extensively analyzed localization mechanisms on a domain wall. In particular, in ref. [43], they proposed a well-defined model for localizing the SM, or something close to it, on a domain wall brane.

4. Discussions

In this paper, we have investigated the possibility of localizing various matter fields on a Weyl thick brane, which also localize the graviton, from the viewpoint of field theory. We first give a brief review of the type of thick smooth brane configuration in a pure geometric Weyl integrable 5-dimensional space time. Then, we check localization of various bulk matter fields on the pure geometrical thick brane and obtain the KK spectrums for the mass-independent potential of these matter fields. When $0 < p < 3/4$, the one dimensional Schrödinger potentials for scalars, vectors and fermions are similar to the one for gravity obtained in ref. [30]. They have a finite negative well at the location of the brane and a finite positive barrier at each side which vanishes asymptotically. It is shown that there is only one single bound state (zero mode) which is just the lowest energy eigenfunction of the Schrödinger equation for the three kinds of fields. Since all values of $m^2 > 0$ are allowed, there also exist a continuum gapless spectrum of KK states with $m^2 > 0$, which turn asymptotically into continuum plane wave as $|z| \to \infty$ [2, 20, 27, 28]. All of these zero modes including the one for spin 1 vectors are normalized and bound, so all these matter fields are localized on the brane.

When $p = 3/4$, the potentials are the modified Pöschl-Teller potentials. They are also similar to the case of gravity and have a finite negative well at the location of the brane and a finite positive barrier at each side which doesn't vanishes. These potentials suggest that there exist mass gap and a series of continuous spectrum starting at positive m^2. The discrete KK modes are bound states while the continuous ones are not. For scalars, there are two bound KK modes, which is just same as the case of gravity. For spin one vectors, there is only one bound state, which is the zero mode. The total number of bound states for spin half fermions is determined by the coupling constant η. For positive coupling

JHEP08(2008)041

constant, the number of bound states of right chiral fermions is one less than that of left chiral fermions. If $0 < \eta \leq 3/8$, there is only one left chiral fermion bound state which is just the left chiral fermion zero mode. If $\eta > 3/8$, there are $N_L (N_L \geq 2)$ left chiral fermion bound states (including zero mode and massive KK modes) and $N_L - 1$ right chiral fermion bound states (only including massive KK modes). For negative coupling constant, we will get similar results but need to interchange left and right, e.g., there is the localized right chiral fermion zero mode but not the localized left one. In the case of no coupling ($\eta = 0$), there are no any localized fermion KK modes including zero modes for both left and right chiral fermions. Hence, for left or right chiral fermions localization, there must be some kind of coupling. These situations can be compared with the case of the domain wall in the RS framework [13], where for localization of spin 1/2 field additional localization method by Jackiw and Rebbi [44] was introduced.

Acknowledgments

It is a pleasure to thank the authors of ref. [27] for their very helpful and interesting discussion. This work was supported by the National Natural Science Foundation of the People's Republic of China (No. 502-041016, No. 10475034 and No. 10705013) and the Fundamental Research Fund for Physics and Mathematics of Lanzhou University (No. Lzu07002).

References

[1] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [hep-ph/9905221]; *An alternative to compactification*, *Phys. Rev. Lett.* **83** (1999) 4690 [hep-th/9906064].

[2] J.D. Lykken and L. Randall, *The shape of gravity*, *JHEP* **06** (2000) 014 [hep-th/9908076].

[3] V.A. Rubakov and M.E. Shaposhnikov, *Do we live inside a domain wall?*, *Phys. Lett.* **B 125** (1983) 136.

[4] V.A. Rubakov and M.E. Shaposhnikov, *Extra space-time dimensions: towards a solution to the cosmological constant problem*, *Phys. Lett.* **B 125** (1983) 139.

[5] K. Akama, *Proceedings of the symposium on gauge theory and gravitation*, Nara, Japan, K. Kikkawa, N. Nakanishi and H. Nariai eds., Springer-Verlag, New York U.S.A. (1983).

[6] M. Visser, *An exotic class of Kaluza-Klein models*, *Phys. Lett.* **B 159** (1985) 22 [hep-th/9910093].

[7] S. Randjbar-Daemi and C. Wetterich, *Kaluza-Klein solutions with noncompact internal spaces*, *Phys. Lett.* **B 166** (1986) 65.

[8] I. Antoniadis, *A Possible new dimension at a few TeV*, *Phys. Lett.* **B 246** (1990) 377.

[9] N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, *The hierarchy problem and new dimensions at a millimeter*, *Phys. Lett.* **B 429** (1998) 263 [hep-ph/9803315];
I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, *New dimensions at a millimeter to a Fermi and superstrings at a TeV*, *Phys. Lett.* **B 436** (1998) 257 [hep-ph/9804398].

[10] A. Kehagias, *A conical tear drop as a vacuum-energy drain for the solution of the cosmological constant problem*, *Phys. Lett.* **B 600** (2004) 133 [hep-th/0406025].

[11] E.J. Squires, *Dimensional reduction caused by a cosmological constant*, *Phys. Lett.* **B 167** (1986) 286.

[12] M. Gogberashvili, *Four dimensionality in non-compact Kaluza-Klein model*, *Mod. Phys. Lett.* **A 14** (1999) 2025 [hep-ph/9904383]; *Hierarchy problem in the shell-universe model*, *Int. J. Mod. Phys.* **D 11** (2002) 1635 [hep-ph/9812296].

[13] B. Bajc and G. Gabadadze, *Localization of matter and cosmological constant on a brane in Anti de Sitter space*, *Phys. Lett.* **B 474** (2000) 282 [hep-th/9912232].

[14] I. Oda, *Localization of matters on a string-like defect*, *Phys. Lett.* **B 496** (2000) 113 [hep-th/0006203].

[15] Y. Grossman and M. Neubert, *Neutrino masses and mixings in non-factorizable geometry*, *Phys. Lett.* **B 474** (2000) 361 [hep-ph/9912408];
S. Ichinose, *Fermions in Kaluza-Klein and Randall-Sundrum theories*, *Phys. Rev.* **D 66** (2002) 104015 [hep-th/0206187];
R. Koley and S. Kar, *A novel braneworld model with a bulk scalar field*, *Phys. Lett.* **B 623** (2005) 244 [*Erratum ibid.* **631** (2005) 199] [hep-th/0507277].

[16] C. Ringeval, P. Peter and J.-P. Uzan, *Localization of massive fermions on the brane*, *Phys. Rev.* **D 65** (2002) 044016 [hep-th/0109194].

[17] R. Koley and S. Kar, *Scalar kinks and fermion localisation in warped spacetimes*, *Class. and Quant. Grav.* **22** (2005) 753 [hep-th/0407158].

[18] T. Gherghetta and M.E. Shaposhnikov, *Localizing gravity on a string-like defect in six dimensions*, *Phys. Rev. Lett.* **85** (2000) 240 [hep-th/0004014].

[19] I.P. Neupane, *Consistency of higher derivative gravity in the brane background*, *JHEP* **09** (2000) 040 [hep-th/0008190]; *Localized gravity with higher curvature terms*, *Class. and Quant. Grav.* **19** (2002) 5507 [hep-th/0106100].

[20] O. DeWolfe, D.Z. Freedman, S.S. Gubser and A. Karch, *Modeling the fifth dimension with scalars and gravity*, *Phys. Rev.* **D 62** (2000) 046008 [hep-th/9909134].

[21] M. Gremm, *Four-dimensional gravity on a thick domain wall*, *Phys. Lett.* **B 478** (2000) 434 [hep-th/9912060]; *Thick domain walls and singular spaces*, *Phys. Rev.* **D 62** (2000) 044017 [hep-th/0002040].

[22] C. Csáki, J. Erlich, T.J. Hollowood and Y. Shirman, *Universal aspects of gravity localized on thick branes*, *Nucl. Phys.* **B 581** (2000) 309 [hep-th/0001033].

[23] R. Emparan, R. Gregory and C. Santos, *Black holes on thick branes*, *Phys. Rev.* **D 63** (2001) 104022 [hep-th/0012100];
S. Kobayashi, K. Koyama and J. Soda, *Thick brane worlds and their stability*, *Phys. Rev.* **D 65** (2002) 064014 [hep-th/0107025];
A. Campos, *Critical phenomena of thick branes in warped spacetimes*, *Phys. Rev. Lett.* **88** (2002) 141602 [hep-th/0111207];
A.-Z. Wang, *Thick de Sitter brane worlds, dynamic black holes and localization of gravity*, *Phys. Rev.* **D 66** (2002) 024024 [hep-th/0201051];
R. Guerrero, A. Melfo and N. Pantoja, *Self-gravitating domain walls and the thin-wall limit*, *Phys. Rev.* **D 65** (2002) 125010 [gr-qc/0202011];

JHEP08(2008)041

A. Melfo, N. Pantoja and A. Skirzewski, *Thick domain wall spacetimes with and without reflection symmetry*, *Phys. Rev.* **D 67** (2003) 105003 [gr-qc/0211081];
K.A. Bronnikov and B.E. Meierovich, *A general thick brane supported by a scalar field*, *Grav. Cosmol.* **9** (2003) 313 [gr-qc/0402030];
O. Castillo-Felisola, A. Melfo, N. Pantoja and A. Ramirez, *Localizing gravity on exotic thick 3-branes*, *Phys. Rev.* **D 70** (2004) 104029 [hep-th/0404083];
M. Minamitsuji, W. Naylor and M. Sasaki, *Quantum fluctuations on a thick de Sitter brane*, *Nucl. Phys.* **B 737** (2006) 121 [hep-th/0508093];
T.R. Slatyer and R.R. Volkas, *Cosmology and fermion confinement in a scalar-field-generated domain wall brane in five dimensions*, *JHEP* **04** (2007) 062 [hep-ph/0609003].

[24] V. Dzhunushaliev, V. Folomeev, D. Singleton and S. Aguilar-Rudametkin, *Thick branes from scalar fields*, *Phys. Rev.* **D 77** (2008) 044006 [hep-th/0703043];
V. Dzhunushaliev, V. Folomeev, K. Myrzakulov and R. Myrzakulov, *Thick brane in 7D and 8D spacetimes*, arXiv:0705.4014.

[25] S. Randjbar-Daemi and M.E. Shaposhnikov, *Fermion zero-modes on brane-worlds*, *Phys. Lett.* **B 492** (2000) 361 [hep-th/0008079].

[26] O. Arias, R. Cardenas and I. Quiros, *Thick brane worlds arising from pure geometry*, *Nucl. Phys.* **B 643** (2002) 187 [hep-th/0202130].

[27] N. Barbosa-Cendejas and A. Herrera-Aguilar, *4D gravity localized in non $Z(2)$-symmetric thick branes*, *JHEP* **10** (2005) 101 [hep-th/0511050].

[28] N. Barbosa-Cendejas and A. Herrera-Aguilar, *Localization of 4D gravity on pure geometrical thick branes*, *Phys. Rev.* **D 73** (2006) 084022 [*Erratum ibid.* **77** (2008) 049901] [hep-th/0603184].

[29] Y.-X. Liu, X.-H. Zhang, L.-D. Zhang and Y.-S. Duan, *Localization of matters on pure geometrical thick branes*, *JHEP* **02** (2008) 067 [arXiv:0708.0065].

[30] N. Barbosa-Cendejas, A. Herrera-Aguilar, M.A. Reyes Santos and C. Schubert, *Mass gap for gravity localized on Weyl thick branes*, *Phys. Rev.* **D 77** (2008) 126013 [arXiv:0709.3552];
N. Barbosa-Cendejas, A. Herrera-Aguilar, U. Nucamendi and I. Quiros, *Mass hierarchy and mass gap on thick branes with Poincaré symmetry*, arXiv:0712.3098.

[31] J.I. Diaz, J. Negro, L.M. Nieto and O. Rosas-Ortiz, *The supersymmetric modified Pöschl-Teller and delta-well potentials*, *J. Phys.* **A32** (1999) 8447.

[32] G.R. Dvali and M.A. Shifman, *Domain walls in strongly coupled theories*, *Phys. Lett.* **B 396** (1997) 64 [*Erratum ibid.* **407** (1997) 64] [hep-th/9612128].

[33] E. Caliceti, V. Grecchi and M. Maioli, *Double wells: Nevanlinna analyticity, distributional Borel sum and asymptotics*, *Commun. Math. Phys.* **176** (1996) 1;
J. Zamastil, J. Cizek and L. Skala, *WKB approach to calculating the lifetime of quasistationary states*, *Phys. Rev. Lett.* **84** (2000) 5683;
F.J. Gomez and L. Sesma, *Time delay and resonances in "volcano" potentials*, *Phys. Letts.* **A301** (2002) 184;
M.M. Nieto, *A simple volcano potential with an analytic, zero-energy, ground state*, *Phys. Lett.* **B 486** (2000) 414.

[34] H. Davoudiasl, J.L. Hewett and T.G. Rizzo, *Bulk gauge fields in the Randall-Sundrum model*, *Phys. Lett.* **B 473** (2000) 43 [hep-ph/9911262].

JHEP08(2008)041

[35] L.D. Landau and E.M. Lifshitz, *Quantum Mechanics*, Pergamon Press, U.K. (1965); S. Flügge, *Practical quantum mechanics*, Springer-Verlag, New York U.S.A. (1974).

[36] A. Melfo, N. Pantoja and J.D. Tempo, *Fermion localization on thick branes*, *Phys. Rev.* **D 73** (2006) 044033 [hep-th/0601161].

[37] S.L. Parameswaran, S. Randjbar-Daemi and A. Salvio, *Gauge fields, fermions and mass gaps in* 6*D brane worlds*, *Nucl. Phys.* **B 767** (2007) 54 [hep-th/0608074].

[38] Y.-X. Liu, L. Zhao and Y.-S. Duan, *Localization of fermions on a string-like defect*, *JHEP* **04** (2007) 097 [hep-th/0701010].

[39] G. De Pol, H. Singh and M. Tonin, *Action with manifest duality for maximally supersymmetric six-dimensional supergravity*, *Int. J. Mod. Phys.* **A 15** (2000) 4447 [hep-th/0003106].

[40] Y.-X. Liu, L. Zhao, X.-H. Zhang and Y.-S. Duan, *Fermions in self-dual vortex background on a string-like defect*, *Nucl. Phys.* **B 785** (2007) 234 [arXiv:0704.2812].

[41] Y.-Q. Wang, T.-Y. Si, Y.-X. Liu and Y.-S. Duan, *Fermionic zero modes in self-dual vortex background*, *Mod. Phys. Lett.* **A 20** (2005) 3045 [hep-th/0508111]; Y.-S. Duan, Y.-X. Liu and Y.-Q. Wang, *Fermionic zero modes in gauge and gravity backgrounds on* T^2, *Mod. Phys. Lett.* **A 21** (2006) 2019 [hep-th/0602157]; Y.-X. Liu, Y.-Q. Wang and Y.-S. Duan, *Fermionic zero modes in self-dual vortex background on a torus*, *Commun. Theor. Phys.* **48** (2007) 675.

[42] R. Jackiw and P. Rossi, *Zero modes of the vortex-fermion system*, *Nucl. Phys.* **B 190** (1981) 681.

[43] R. Davies, D.P. George and R.R. Volkas, *The standard model on a domain-wall brane*, *Phys. Rev.* **D 77** (2008) 124038 [arXiv:0705.1584].

[44] R. Jackiw and C. Rebbi, *Solitons with fermion number* 1/2, *Phys. Rev.* **D 13** (1976) 3398.

JHEP08(2008)041